

Score:

Name:

Solutions

Period (circle one): 1 2 3 4 5 6

Team (circle one): a b c d e f

### SM286 – Quiz 15

### Systems of DEs/Eigenvalues/Eigenvectors

1. Solve the following system of D.E.'s :

$$\begin{cases} x' = 4x + 2y \\ y' = x + 3y \end{cases}, \quad \begin{matrix} x(0) = 0 \\ y(0) = 3 \end{matrix}$$

Using your solutions, calculate  $x(1)$  and  $y(1)$  to 2 decimal places.

$$X' = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix} X \Rightarrow \det \begin{bmatrix} 4-\lambda & 2 \\ 1 & 3-\lambda \end{bmatrix} = 12 - 7\lambda + \lambda^2 - 2$$

$$\Rightarrow (\lambda^2 - 7\lambda + 10) = 0 \Rightarrow \lambda = 2, 5$$
$$(\lambda - 5)(\lambda - 2) = 0$$

$$\Rightarrow \lambda = 5 \Rightarrow \begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow -x_1 + 2x_2 = 0 \Rightarrow x_1 = 2x_2$$

$$\boxed{V_{\lambda=5} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}}$$

$$\Rightarrow \lambda = 2 \Rightarrow \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x_1 = -x_2 \Rightarrow V_{\lambda=2} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\Rightarrow X = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{5t} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{2t}$$

$$\Rightarrow x = 2c_1 e^{5t} + c_2 e^{2t}$$

$$y = c_1 e^{5t} - c_2 e^{2t}$$

$$x(0) = 2c_1 + c_2 = 0$$

$$y(0) = \frac{c_1 - c_2 = 3}{3c_1 = 3} \Rightarrow \boxed{c_1 = 1}$$

$$\boxed{c_2 = -2}$$

$$\Rightarrow \begin{cases} x = 2e^{5t} - 2e^{2t} \\ y = e^{5t} + 2e^{2t} \end{cases}$$