Score:

Name: Solutions
Period (circle one): 1 2 3 4 5 6
Team (circle one): a b c d e f

SM286 – Quiz 18
Power Series Solutions to DEs

1. Solve the DE $x' - x = 0$ using power series solutions.
   a. Write the solution in the form of a general summation..
   b. Does the general form look like something that you have seen before?

$$x = \sum_{n=0}^{\infty} c_n t^n \Rightarrow \quad x' = \sum_{n=1}^{\infty} c_n n t^{n-1}$$

$$\Rightarrow \sum_{n=1}^{\infty} c_n n t^{n-1} - \sum_{n=0}^{\infty} c_n t^n = 0$$

$k = n-1$
$n = k+1$
$n=1 \Rightarrow k=0$

$n = k$

$$\sum_{k=0}^{\infty} c_{k+1}(k+1)t^k - \sum_{k=0}^{\infty} c_k t^k = 0 \Rightarrow \sum_{n=0}^{\infty} \left((c_{k+1})(k+1) - c_k\right) x^k = 0$$

$$\Rightarrow \quad c_{k+1} = \frac{c_k}{k+1}$$

$$x = c_0 + \frac{c_0}{1!}t + \frac{c_0}{2!}t^2 \ldots$$

$$\Rightarrow \quad k=0 \Rightarrow c_1 = \frac{c_0}{1}$$

$$k=1 \Rightarrow c_2 = \frac{c_1}{2} = \frac{c_0}{2}$$

$$k=2 \Rightarrow c_3 = \frac{c_2}{3} = \frac{c_0}{6}$$

$$k=3 \Rightarrow c_4 = \frac{c_3}{4} = \frac{c_0}{24}$$

$$k=n = \frac{c_0}{n!}$$

$$\boxed{x = c_0 \sum_{n=0}^{\infty} \frac{t^n}{n!}}$$

↑ Looks like Maclaurin series

$$\boxed{x = c_0 e^t}$$