Score:

Name: Solutions
Period (circle one): 1 2 3 4 5 6
Team (circle one): a b c d e f

SM286 – Diff. Eq – Quiz 20 – Sections 11.1
Orthogonal Functions

1. Let $f(x)$ and $g(x)$ be orthogonal functions on $[a, b]$. Show that:

$$\|f(x) + g(x)\|^2 = \|f(x)\|^2 + \|g(x)\|^2$$

(Recall the definition of the norm of a function: $\|\emptyset(x)\|^2 = \int_a^b \emptyset(x)\emptyset(x)dx = \int_a^b [\emptyset(x)]^2 dx$).

$$\|f(x)+g(x)\|^2 = \int_a^b (f(x)+g(x))(f(x)+g(x))dx$$

$$= \int_a^b [f(x)]^2 + 2f(x)g(x) + [g(x)]^2 dx$$

$$= \int_a^b [f(x)]^2 dx + 2\int_a^b f(x)g(x)dx + \int_a^b [g(x)]^2 dx$$

$$\|f(x)\|^2 + 0 + \|g(x)\|^2$$

definition of norm

$f(x)$ & $g(x)$ orthogonal

definition of norm

$$\therefore \boxed{\|f(x)+g(x)\|^2 = \|f(x)\|^2 + \|g(x)\|^2}$$