Score:

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SM286 – Quiz 21 – Sections 11.2
Fourier Series

1. Find the Fourier Series for $f(x) = \begin{cases} x^2 & -1 < x < 0 \\ 0 & 0 \le x \le 1 \end{cases}$. Sketch the Fourier Series as the number of terms approach infinity.

Hint: The Fourier series of a function $f(x)$ defined on the interval $(-p, p)$ is given by:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n cos\left(\frac{n\pi x}{p}\right) + b_n \sin\left(\frac{n\pi x}{p}\right) \right)$$

Where:

$$a_o = \frac{1}{p}\int_{-p}^{p} f(x)dx$$

$$a_n = \frac{1}{p}\int_{-p}^{p} f(x)cos\left(\frac{n\pi x}{p}\right)dx$$

$$b_n = \frac{1}{p}\int_{-p}^{p} f(x)sin\left(\frac{n\pi x}{p}\right)dx$$

$a_0 = \int_{-1}^{0} x^2\,dx = \frac{1}{3}x^3\Big|_{-1}^{0} = \left(0 - \frac{-1}{3}\right) = \frac{1}{3}$ ⇒

$a_n = \int_{-1}^{0} x^2 \cos(n\pi x)\,dx = \frac{2(-1)^n}{n^2\pi^2}$

$b_n = \int_{-1}^{0} x^2 \sin(n\pi x)\,dx = \frac{(-1)^n}{n\pi} - \frac{2(-1)^n}{n^3\pi^3} + \frac{2}{n^3\pi^3}$

⇒ $f(x) = \frac{1}{6} + \sum_{n=1}^{\infty} \frac{2(-1)^n}{n^2\pi^2}\cos(n\pi x) + \left[\frac{(-1)^n}{n\pi} + \frac{2}{n^3\pi^3}\left(1-(-1)^n\right)\right]\sin(n\pi x)$