

Score:

Name:

Solutions

Period (circle one): 1 2 3 4 5 6

Team (circle one): a b c d e f

SM286 – Quiz 22 – Sections 11.3 Fourier Sine/Cosine Series

1. Find the Fourier sine series for $f(x) = x^2$ $0 \leq x \leq \pi$. Sketch the sine series as the number of terms go to infinite.

Hint: The Fourier sine series of a function $f(x)$ defined on the interval $[0, p]$ is given by:

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{p}\right)$$

Where:

$$b_n = \frac{2}{p} \int_0^p f(x) \sin\left(\frac{n\pi x}{p}\right) dx$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} x^2 \sin(nx) dx = \frac{2}{\pi} \left[-\frac{(-1)^n \pi^2}{n} + \frac{2(-1)^n}{n^3} - \frac{2}{n^3} \right]$$
$$= 2 \left[\frac{(-1)^n \pi}{n} + \frac{2}{n^3 \pi} [(-1)^n - 1] \right]$$

$$\Rightarrow f(x) \approx 2 \sum_{n=1}^{\infty} \left[\frac{(-1)^{n+1} \pi}{n} + \frac{2}{n^3 \pi} [(-1)^n - 1] \right] \sin(nx)$$

